The Total Belief Theorem

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Suppose $\Theta$ and $\Omega$ are two frames of discernment, and $\rho : 2^{\Omega} \rightarrow 2^{\Theta}$ a given refining between them. Let $b_0$ be a belief function defined over $\Omega = \{\omega_1, ..., \omega_{|\Omega|}\}$. Suppose there exists a collection of belief functions $b_i : 2^{\Pi_i} \rightarrow [0, 1]$, where $\Pi = \{\Pi_1, ..., \Pi_{|\Omega|}\}$, $\Pi_i = \rho(\{\omega_i\})$, is the partition of $\Theta$ induced by its coarsening $\Omega$. Then, there exists a total belief function $b : 2^\Theta \rightarrow [0, 1]$ such that:

- (P1) $b \odot b_{\Pi_i} = b_i \ \forall i = 1, ..., |\Omega|$, where $b_{\Pi_i}$ is the categorical belief function with mass $m_{\Pi_i}$;
- (P2) $b_0$ is the marginal of $b$ on $\Omega$, $b_0 = b \upharpoonright_{\Omega}$.