

The Total Belief Theorem

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Suppose Θ and Ω are two frames of discernment, and $\rho : 2^\Omega \rightarrow 2^\Theta$ a given refining between them. Let b_0 be a belief function defined over $\Omega = \{\omega_1, \dots, \omega_{|\Omega|}\}$. Suppose there exists a collection of belief functions $b_i : 2^{\Pi_i} \rightarrow [0, 1]$, where $\Pi = \{\Pi_1, \dots, \Pi_{|\Omega|}\}$, $\Pi_i = \rho(\{\omega_i\})$, is the partition of Θ induced by its coarsening Ω . Then, there exists a total belief function $b : 2^\Theta \rightarrow [0, 1]$ such that:

- ▶ (P1) $b \oplus b_{\Pi_i} = b_i \ \forall i = 1, \dots, |\Omega|$, where b_{Π_i} is the categorical belief function with mass m_{Π_i} ;
- ▶ (P2) b_0 is the marginal of b on Ω , $b_0 = b \upharpoonright_\Omega$.

